

Exact fermion pair creation by an electric field in dS₂ space-time

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The effect of the electric field on the creation of spin 1/2 particles from vacuum in the (1+1) dimensional de-Sitter space-time is studied. The Dirac equation with a constant electric field is solved by introducing an unitary transformation. Then the canonical method based on Bogoliubov transformation is applied to calculate the pair creation probability and the density of created particles both for positive or negative wave vector. It is shown that in the presence of a constant electric field the density of created particles is more significant when the wave vector is in a well-defined direction. This depends on the orientation of the electric field and the sign of the particle charge. The limit $H = 0$, where dS space reduces to the flat Minkowski space-time, is studied. The imaginary part of the effective action is extracted from the vacuum to vacuum transition amplitude.

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I. INTRODUCTION

As we know our universe is undergoing an accelerated expansion that can be approximated by an exponentially expanding de-Sitter space-time. It is widely believed, also, that the de-Sitter space-time allows a better description of the early stage of inflation. Therefore de-Sitter metric is of interest both for the cosmology of the early and late universe. This importance comes also from the fact that the de-Sitter space is the unique maximally symmetric curved space and enjoys the same degrees of symmetry as Minkowski space-time. This explains the increased interest to study physical quantum effects in such a space-time. Among these effects we cite the particle-antiparticle pair creation caused by the expansion of the universe [1–5]. The phenomenon of particle creation in de-Sitter space has been widely discussed and analyzed from various point of view [6–15]. This is because the fact that the particle creation has many important applications in contemporary cosmology - e.g., it could have consequences for early universe cosmology and may play an important role in the exit from inflationary universe and in the cosmic evolution [16–21]. Also, the particle creation mechanism has significant contribution on the transition from an anisotropic universe to an isotropic one [22].

The purpose of this paper is to study the effect of the electric field on the creation of fermions from vacuum in the de-Sitter space-time. We know that actually there is no electric field in the universe. However, as is mentioned in [23], we reconcile that electric fields were present during the initial stages of the formation of the universe and they vanish because

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of the inverse effect of the particle creation. This idea resembles to the assumption of the anisotropy of the early universe although the present universe is isotropic. Then the existence of such electric fields influences the cosmic evolution directly via Friedmann equations and by their effect on the creation of particles.

We note that the influence of electric field on the particle creation in expanding universe has been studied in several scenarios describing different stages of the cosmic evolution [12, 22–28]. The creation of spin half fermions in (1+1) dimensional de-Sitter space-time by an electric field has been communicated several years ago in [29] where the author has found the positive and negative energy solutions for the corresponding Dirac equation. He has not shown however how those states are connected to one another by the charge conjugation. This symmetry between positive and negative solutions is indispensable in the derivation of the effect.

In order to study the effect of particle-antiparticle pair creation from vacuum by gravitational and electric fields we have at our disposal several methods such as the adiabatic method [30, 31], the Hamiltonian diagonalization technique [32, 33], the Feynman path integral derivation [34, 35], the Green function approach [36, 37], the semiclassical WKB approximation [38–40] as well as the method based on Bogoliubov transformation [41, 42] that we shall use in this paper. The later method is more convenient for the spin $\frac{1}{2}$ case.

The paper is organized as follows; At the beginning we consider a spin $\frac{1}{2}$ fermion subjected to an electric field in the (1 + 1) dimensional de-Sitter space-time. Then we solve the corresponding Dirac equation by introducing an unitary transformation. To investigate the process of particle creation we apply the canonical method based on Bogoliubov transformation connecting the "in" with the "out" states. This method permits us to determine the probability of pair creation, the density number of created particles and the vacuum persistence. To complete the analysis we write the probability of pair production per unit of time as a Schwinger series starting from the vacuum to vacuum transition amplitude and we show how the electric field amplifies particle creation.

II. DIRAC EQUATION IN DE-SITTER SPACE

Let us consider a spin $\frac{1}{2}$ fermion of mass m and charge $-e$ moves on the background geometry of the (1 + 1) dimensional de-Sitter space-time in the presence of an electric field. The line element of the metric describing dS_2 space-time can be written as

$$ds^2 = dt^2 - \tilde{a}^2(t) dx^2, \quad (1)$$

where $\tilde{a}(t) = e^{Ht}$ and H is the Hubble's constant. In terms the conformal time $\eta = \int dt/a(t) = \frac{-1}{H}e^{-Ht}$ this metric reads

$$ds^2 = a^2(\eta) (d\eta^2 - dx^2) \quad (2)$$

where the new scale factor $a(\eta) \equiv \tilde{a}[t(\eta)] = \frac{-1}{H\eta}$. For the electric field we choose the gauge $A_\mu = (0, A_1(\eta))$, with

$$A_1 = -\frac{E_0}{H^2\eta}. \quad (3)$$

This vector potential describes a constant electric field in the comoving system of coordinates.

The covariant Dirac equation with the metric (2) takes the form

$$[i\tilde{\gamma}^\mu(\eta)(\partial_\mu - ieA_\mu(\eta) - \Gamma_\mu(\eta)) - m]\psi = 0 \quad (4)$$

where the curvature-dependent Dirac matrices $\tilde{\gamma}^\mu(\eta)$ are given in diagonal tetrad gauge by

$$\tilde{\gamma}^0(\eta) = \frac{1}{a(\eta)}\gamma^0 \quad (5)$$

$$\tilde{\gamma}^1(\eta) = -\frac{1}{a(\eta)}\gamma^1 \quad (6)$$

where γ^0 and γ^1 are the usual Dirac's matrices that can be written, in (1+1) dimensional Minkowski space, in terms of Pauli matrices as follows

$$\gamma^0 = \sigma_z, \quad \gamma^1 = i\sigma_y \quad (7)$$

and $\Gamma_\mu(\eta)$ are the spin connections

$$\begin{aligned} \Gamma_0 &= 0 \\ \Gamma_1 &= \frac{1}{2} \frac{\dot{a}(\eta)}{a(\eta)} \gamma_0 \gamma_1. \end{aligned} \quad (8)$$

By making the substitution $\chi(\eta, x) = a^{\frac{1}{2}}(\eta)\psi(\eta, x)$, we obtain the simpler equation

$$[\gamma^\mu(i\partial_\mu + eA_\mu(\eta)) - ma(\eta)]\chi(\eta, x) = 0. \quad (9)$$

In order to solve this equation we write, at the beginning, $\chi(\eta, x) = \exp(ikx)\xi(\eta)$ with

$$\xi(\eta) = \begin{pmatrix} \xi_1(\eta) \\ \xi_2(\eta) \end{pmatrix} \quad (10)$$

where the two components $\xi_1(\eta)$ and $\xi_2(\eta)$ satisfy the two coupled equations

$$\left(i\frac{\partial}{\partial\eta} + \frac{m}{H\eta}\right)\xi_1(\eta) = \left(k + \frac{eE_0}{H^2\eta}\right)\xi_2(\eta) \quad (11)$$

$$\left(i\frac{\partial}{\partial\eta} - \frac{m}{H\eta}\right)\xi_2(\eta) = \left(k + \frac{eE_0}{H^2\eta}\right)\xi_1(\eta). \quad (12)$$

Here we remark that the coupling coefficient $k + \frac{eE_0}{H^2\eta}$ depends on the conformal time and the usual iteration procedure leads to a complicated second order equation that does not admit well-known solutions. To simplify the problem let us introduce the unitary transformation

$$\begin{pmatrix} \xi_1(\eta) \\ \xi_2(\eta) \end{pmatrix} = \frac{1}{\sqrt{1+\tau^2}} \begin{pmatrix} 1 & \tau \\ -\tau & 1 \end{pmatrix} \begin{pmatrix} \varphi_1(\eta) \\ \varphi_2(\eta) \end{pmatrix} \quad (13)$$

with

$$\tau = \frac{H}{eE_0}(\mathcal{M} - m) \quad (14)$$

and

$$\mathcal{M} = \sqrt{m^2 + \frac{e^2 E_0^2}{H^2}}. \quad (15)$$

Then the novel components $\varphi_1(\eta)$ and $\varphi_2(\eta)$ satisfy the following system of equations

$$\left[i \frac{\partial}{\partial \eta} + \frac{1}{\mathcal{M}H} \left(eE_0 k + \frac{\mathcal{M}^2}{\eta} \right) \right] \varphi_1 = \frac{m}{\mathcal{M}} k \varphi_2 \quad (16)$$

$$\left[i \frac{\partial}{\partial \eta} - \frac{1}{\mathcal{M}H} \left(eE_0 k + \frac{\mathcal{M}^2}{\eta} \right) \right] \varphi_2 = \frac{m}{\mathcal{M}} k \varphi_1 \quad (17)$$

which leads to a second-order equation for each component

$$\left[\frac{\partial^2}{\partial \eta^2} + \left(\frac{\mathcal{M}^2}{H^2} \pm i \frac{\mathcal{M}}{H} \right) \frac{1}{\eta^2} + \frac{2eE_0 k}{H^2} \frac{1}{\eta} + k^2 \right] \varphi_{1,2} = 0. \quad (18)$$

Let us notice that the gravitational field couples to the mass of the particle, while the electric field couples to the charge. In the novel system we can see that an effective field is coupled to the quantity \mathcal{M} . As we will show, for the process of particle creation, this quantity is more important than the mass of the particle. In effect, writing the equation (18) in the form $\varphi_s''(\eta) + \omega_s^2(\eta)\varphi_s(\eta) = 0$, with $\omega_s^2(\eta) = \left(\frac{\mathcal{M}^2}{H^2} + i s \frac{\mathcal{M}}{H} \right) \frac{1}{\eta^2} + 2 \frac{eE_0 k}{H^2} \frac{1}{\eta} + k^2$, we can see that the adiabatic condition implies that

$$\lim_{\eta \rightarrow 0} \left| \frac{\dot{\omega}_s(\eta)}{\omega_s^2(\eta)} \right| \approx \frac{H}{\mathcal{M}} \ll 1 \quad (19)$$

Therefore the particle production is well-defined only when $\mathcal{M} \gg H$. Furthermore, in the limit $m \rightarrow 0$, the mixing term in (16) and (17) vanishes and the positive and negative energy solutions never intercept each other. This means that there is no production of massless particles even if an electric field is present.

In order to solve equation (18) we make the change of variable $\eta \rightarrow \rho$, where

$$\rho = -2ik\eta. \quad (20)$$

The resulting equation is similar to the well-known Whittaker equation [43]

$$\left[\frac{d^2}{d\rho^2} - \frac{1}{4} + \frac{\lambda}{\rho} + \frac{\frac{1}{4} - \mu_s^2}{\rho^2} \right] \tilde{\varphi}_s(\rho) = 0, \quad (21)$$

with $\tilde{\varphi}_s(\rho) \equiv \varphi_s(\eta)$, $s = \overline{1, 2}$ and the constants μ_s^2 and λ are given by

$$\begin{aligned} \mu_1 &= \frac{1}{2} - i \frac{\mathcal{M}}{H} = \mu \\ \mu_2 &= \frac{1}{2} + i \frac{\mathcal{M}}{H} = \mu^* = 1 - \mu \end{aligned} \quad (22)$$

and

$$\lambda = i \frac{eE_0}{H^2}. \quad (23)$$

It is known that one can find for the equation (21) several sets of linearly independent solutions which can be written in terms of the Whittaker functions $M_{\lambda, \mu}(\rho)$ and $W_{\lambda, \mu}(\rho)$, with

$$M_{\lambda, \mu}(\rho) = \rho^{\mu + \frac{1}{2}} e^{-\frac{\rho}{2}} M \left(\mu - \lambda + \frac{1}{2}, 2\mu + 1; \rho \right) \quad (24)$$

$$W_{\lambda, \mu}(\rho) = \rho^{\mu + \frac{1}{2}} e^{-\frac{\rho}{2}} U \left(\mu - \lambda + \frac{1}{2}, 2\mu + 1; \rho \right), \quad (25)$$

where $M(a, b, \rho)$ and $U(a, b, \rho)$ are the Kummar functions [44].

The general technique for solving the Dirac equation in (1+1) dimensional de-Sitter space-time when an electric field is present being shown, let us use the obtained solutions to investigate the phenomenon of spin half particle creation.

III. VACUUM DEFINITION AND PAIR CREATION

The study of particle creation in an expanding universe requires a definition of a vacuum state for the field theory [45]. However, unlike the free Dirac field, it is not obvious how to determine the vacuum states when the spinor field is subjected to a general gravitational background. It is widely believed that in arbitrary curved background, there is no absolute definition of the vacuum state and the concept of particles is not completely clear. From physical point of view it is well known that in the standard quantum theory a particle cannot be localized to a region smaller than its de Broglie wavelength. When this wavelength is sufficiently large, the concept particle becomes unclear [46]. Moreover, when the vacuum state is defined in the remote past it is habitually unstable so that it may differ from the vacuum state in the remote future. This gives rise to spontaneous particle creation.

To obtain a well-defined vacuum state with a reasonable choice of positive and negative frequency modes we use the so-called adiabatic method based on the solutions of the relativistic Hamilton-Jacobi equation. Taking into account the asymptotic behavior of the $W_{\lambda,\mu}(\rho)$ functions

$$W_{\lambda,\mu}(\rho) \sim e^{-\frac{\rho}{2}} (-\rho)^\lambda \quad (26)$$

and using the solutions of the Hamilton-Jacobi equation we can find for the "in" states the following components

$$\tilde{\varphi}_{1,in}^+(\rho) = W_{-\lambda,\mu}(-\rho) \quad (27)$$

$$\tilde{\varphi}_{1,in}^-(\rho) = W_{\lambda,\mu}(\rho). \quad (28)$$

By the use of the relation [47]

$$\left[(2\mu - 1) \frac{\partial}{\partial \rho} + \frac{(2\mu - 1)^2}{2\rho} - \lambda \right] W_{\lambda,\mu}(\rho) = - \left(\mu + \lambda - \frac{1}{2} \right) W_{\lambda,\mu-1}(\rho) \quad (29)$$

we find the lower component for the positive and negative frequency modes

$$\tilde{\varphi}_{2,in}^+(\rho) = \frac{1}{m} \left(\mathcal{M} + \frac{eE_0}{H} \right) W_{-\lambda,1-\mu}(-\rho) \quad (30)$$

$$\tilde{\varphi}_{2,in}^-(\rho) = -\frac{1}{m} \left(\mathcal{M} - \frac{eE_0}{H} \right) W_{\lambda,1-\mu}(\rho) \quad (31)$$

For the Dirac spinor we have

$$\xi_{in}^+(\eta) = \mathcal{N} \begin{pmatrix} \sqrt{\mathcal{M} - \frac{eE_0}{H}} W_{-\lambda,\mu}(-\rho) + \tau \sqrt{\mathcal{M} + \frac{eE_0}{H}} W_{-\lambda,1-\mu}(-\rho) \\ -\tau \sqrt{\mathcal{M} - \frac{eE_0}{H}} W_{-\lambda,\mu}(-\rho) + \sqrt{\mathcal{M} + \frac{eE_0}{H}} W_{-\lambda,1-\mu}(-\rho) \end{pmatrix} \quad (32)$$

and

$$\xi_{in}^-(\eta) = \mathcal{N}^* \begin{pmatrix} \sqrt{\mathcal{M} + \frac{eE_0}{H}} W_{\lambda,\mu}(\rho) - \tau \sqrt{\mathcal{M} - \frac{eE_0}{H}} W_{\lambda,1-\mu}(\rho) \\ \tau \sqrt{\mathcal{M} + \frac{eE_0}{H}} W_{\lambda,\mu}(\rho) + \sqrt{\mathcal{M} - \frac{eE_0}{H}} W_{\lambda,1-\mu}(\rho) \end{pmatrix} \quad (33)$$

where \mathcal{N} is a normalization constant that is unimportant vis-a-vis the mechanism of particle creation. Note that we have multiplied $\xi_{in}^+(\eta)$ by $\sqrt{\mathcal{M} - \frac{eE_0}{H}}$ and $\xi_{in}^-(\eta)$ by $\sqrt{\mathcal{M} + \frac{eE_0}{H}}$ to assure that those states are connected to one another by the charge conjugation transformation defined by

$$\xi \rightarrow \xi^c = \sigma_1 \xi^*. \quad (34)$$

In addition positive and negative energy solutions satisfy the orthogonality condition

$$\bar{\xi}^+ \xi^- = \bar{\xi}^- \xi^+ = 0. \quad (35)$$

With this choice we have a well-defined vacuum state which is no thing but the adiabatic vacuum.

Let us now define the "out" states. As previously, by taking into account the asymptotic behavior of $M_{\lambda,\mu}(\rho)$ at $\rho \rightarrow 0$,

$$M_{\lambda,\mu}(\rho) \sim e^{-\frac{\rho}{2}} \rho^{\mu+\frac{1}{2}}, \quad (36)$$

we can define the following positive and negative frequency modes

$$\tilde{\varphi}_{1,out}^+(\rho) = M_{\lambda,-\mu}(\rho) \quad (37)$$

$$\tilde{\varphi}_{1,out}^-(\rho) = M_{-\lambda,\mu}(-\rho). \quad (38)$$

Then by the use of the functional relation [47]

$$\left[(2\mu+1) \frac{\partial}{\partial \rho} - \frac{(2\mu+1)^2}{2\rho} + \lambda \right] M_{\lambda,\mu}(\rho) = \frac{(\mu + \frac{1}{2})^2 - \lambda^2}{2(\mu+1)(2\mu+1)} M_{\lambda,\mu+1}(\rho) \quad (39)$$

we obtain

$$\varphi_{2,out}^+ = \frac{\frac{m}{\mathcal{M}}}{4(\frac{1}{2} + i\frac{\mathcal{M}}{H})} M_{\lambda,-\mu+1}(\rho). \quad (40)$$

Consequently, we arrive at the following expressions for the two components of the Dirac spinor

$$\xi_{1,out}^+(\eta) = \mathcal{N}' \left[M_{\lambda,-\mu}(\rho) + \tau \frac{\frac{m}{\mathcal{M}}}{4(\frac{1}{2} + i\frac{\mathcal{M}}{H})} M_{\lambda,-\mu+1}(\rho) \right] \quad (41)$$

and

$$\xi_{2,out}^+(\eta) = \mathcal{N}' \left[-\tau M_{\lambda,-\mu}(\rho) + \frac{\frac{m}{\mathcal{M}}}{4(\frac{1}{2} + i\frac{\mathcal{M}}{H})} M_{\lambda,-\mu+1}(\rho) \right]. \quad (42)$$

Using the fact that positive and negative energy solutions must be connected to one another by the charge conjugation we find that

$$\xi_{1,out}^-(\eta) = \mathcal{N}'^* \left[-\tau M_{-\lambda,-1+\mu}(-\rho) + \frac{\frac{m}{\mathcal{M}}}{4(\frac{1}{2} - i\frac{\mathcal{M}}{H})} M_{-\lambda,\mu}(-\rho) \right] \quad (43)$$

and

$$\xi_{2,out}^-(\eta) = \mathcal{N}'^* \left[M_{-\lambda,-1+\mu}(-\rho) + \tau \frac{\frac{m}{\mathcal{M}}}{4 \left(\frac{1}{2} - i \frac{\mathcal{M}}{H} \right)} M_{-\lambda,\mu}(-\rho) \right] \quad (44)$$

Now, as we have mentioned above, in order to determine the probability of pair creation and the density of created particles we use the Bogoliubov transformation connecting the "in" with the "out" states. The relation between those states can be obtained by the use of the relation between Whittaker functions [43]

$$M_{\lambda,\mu}(\rho) = \left[\frac{\Gamma(2\mu+1) e^{-i\pi\lambda}}{\Gamma(\mu-\lambda+\frac{1}{2})} W_{-\lambda,\mu}(-\rho) + \frac{\Gamma(2\mu+1) e^{i\pi(\mu-\lambda+\frac{1}{2})}}{\Gamma(\mu+\lambda+\frac{1}{2})} W_{\lambda,\mu}(\rho) \right], \quad (45)$$

with $-\frac{\pi}{2} < \arg \rho < \frac{3\pi}{2}$ and $2\mu \neq -1, -2, \dots$. The functions $\xi_{out}^+(\eta)$ and $\xi_{out}^-(\eta)$ can be then expressed in terms of $\xi_{in}^+(\eta)$ and $\xi_{in}^-(\eta)$ as follows

$$\begin{aligned} \xi_{out}^+(\eta) &= \alpha \xi_{in}^+(\eta) + \beta \xi_{in}^-(\eta) \\ \xi_{out}^-(\eta) &= \alpha^* \xi_{in}^-(\eta) + \beta^* \xi_{in}^+(\eta) \end{aligned} \quad (46)$$

where the Bogoliubov coefficients α and β are given by

$$\frac{\beta}{\alpha} = \frac{\mathcal{N}}{\mathcal{N}^*} \frac{\Gamma(\frac{1}{2} - \mu - \lambda) \sqrt{\mathcal{M} - \frac{eE_0}{H}}}{\Gamma(\frac{1}{2} - \mu + \lambda) \sqrt{\mathcal{M} + \frac{eE_0}{H}}} e^{i\pi(-\mu+\frac{1}{2})} \quad (47)$$

and

$$|\alpha|^2 + |\beta|^2 = 1. \quad (48)$$

The Bogoliubov relation between "in" and "out" states can be projected in the Fock space to be a relation between creation and annihilation operators

$$\begin{aligned} \hat{b}_{-k,in}^+ &= \alpha^* \hat{b}_{-k,out}^+ + \beta \hat{a}_{k,out} \\ \hat{a}_{k,in} &= \alpha \hat{a}_{k,out} + \beta^* \hat{b}_{-k,out}^+ \end{aligned} \quad (49)$$

Therefore, the probability of pair creation and the density of created particles will be given in terms of Bogoliubov coefficients. In effect starting from the amplitude $\mathcal{A} = \langle 0_{out} | a_{k,out} b_{-k,out} | 0_{in} \rangle$, it is easy to show that

$$\mathcal{A} = -\frac{\beta}{\alpha^*} \langle 0_{out} | 0_{in} \rangle. \quad (50)$$

The probability to create one pair of fermions in the state k is then

$$\mathcal{P}_k = \left| \frac{\beta}{\alpha} \right|^2. \quad (51)$$

By the use of the following property of gamma function [43]

$$|\Gamma(ix)|^2 = \frac{\pi}{x \sinh \pi x} \quad (52)$$

we find

$$\left| \frac{\beta}{\alpha} \right|^2 = \frac{\sinh \pi \left(\frac{\mathcal{M}}{H} + \frac{eE_0}{H^2} \right)}{\sinh \pi \left(\frac{\mathcal{M}}{H} - \frac{eE_0}{H^2} \right)} e^{-2\pi \frac{\mathcal{M}}{H}}. \quad (53)$$

Notice that equation (53) differs from equation (45) in [29]. This is because the author of [29] used an erroneous formula between Whittaker functions (see equation (42) in [29]).

For the density number of created particles we have

$$n(k) = \langle 0_{in} | a_{k,out}^+ a_{k,out} | 0_{in} \rangle = |\beta|^2. \quad (54)$$

Taking into account that the Bogoliubov coefficients satisfy the condition (48), it is easy to show that

$$n(k) = \frac{\sinh \pi \left(\frac{\mathcal{M}}{H} + \frac{eE_0}{H^2} \right) e^{-2\pi \frac{\mathcal{M}}{H}}}{\sinh \pi \left(\frac{\mathcal{M}}{H} - \frac{eE_0}{H^2} \right) + \sinh \pi \left(\frac{\mathcal{M}}{H} + \frac{eE_0}{H^2} \right) e^{-2\pi \frac{\mathcal{M}}{H}}}. \quad (55)$$

Let us note that this result is obtained for a positive wave vector ($k > 0$) and $-\frac{\pi}{2} < \arg \rho < \frac{3\pi}{2}$. For the case when $k < 0$, the quantities $n(k)$ and \mathcal{P}_k can be obtained from (53) and (55) by changing the sign of e . Since the particle creation is well-defined only in the adiabatic limit, $\mathcal{M} \gg H$, the density of created particles can be approximated by

$$n(k) = \exp \left[-\frac{2\pi}{H} \left(\mathcal{M} - \text{sign}(k) \frac{eE_0}{H} \right) \right] \quad (56)$$

which resembles to the Boltzmann distribution. This explains the thermal nature of the effect. In addition it follows from (56) that $n(k)$ is more significant when $k > 0$. Therefore the constant electric field produces predominantly particles with $k > 0$. In other words, in the presence of a constant electric field, particles prefer to be created in a specific direction. This depends on the orientation of the electric field and the sign of the particle charge. The antiparticles are in the main created in the opposite direction with the same density as the particles.

At the end of this section let us examine the limit $H \rightarrow 0$ where the de-Sitter space reduces to the flat Minkowski space-time. Taking into account that $\frac{\mathcal{M}}{H} - \frac{eE_0}{H^2} \approx \frac{m^2}{2eE_0}$, for small H , we easily obtain

$$\mathcal{P}_k \approx \frac{\exp \left[-\pi \left(\frac{\mathcal{M}}{H} - \frac{eE_0}{H^2} \right) \right]}{2 \sinh \pi \left(\frac{\mathcal{M}}{H} - \frac{eE_0}{H^2} \right)} \approx \frac{\exp \left(-\pi \frac{m^2}{eE_0} \right)}{1 - \exp \left(-\pi \frac{m^2}{eE_0} \right)} \quad (57)$$

and

$$n(k) \approx \exp \left(-\pi \frac{m^2}{eE_0} \right). \quad (58)$$

We remark here that equations (57) and (58) are in complete agreements with the well-known results corresponding to the pair creation by an electric field in Minkowski space-time (see [48] and references therein).

IV. SCHWINGER EFFECTIVE ACTION

Let us now use the obtained results to calculate the imaginary part of the Schwinger effective action. We start by writing \mathcal{P}_k as

$$\mathcal{P}_k = \frac{\sigma}{1 - \sigma}, \quad (59)$$

where

$$\begin{aligned}\sigma &= \frac{\sinh \pi \left(\frac{\mathcal{M}}{H} + \epsilon \frac{eE_0}{H^2} \right) e^{-2\pi \frac{\mathcal{M}}{H}}}{\sinh \pi \left(\frac{\mathcal{M}}{H} + \epsilon \frac{eE_0}{H^2} \right) e^{-2\pi \frac{\mathcal{M}}{H}} + \sinh \pi \left(\frac{\mathcal{M}}{H} - \epsilon \frac{eE_0}{H^2} \right)} \\ &\approx \exp \left[-\frac{2\pi}{H} \left(\mathcal{M} - \epsilon \frac{eE_0}{H} \right) \right],\end{aligned}\quad (60)$$

where $\epsilon = \text{sign}(k)$. Let \mathcal{C}_k to be the probability to have no pair creation in the state k . The quantity $\mathcal{C}_k \mathcal{P}_k$ is then the probability to have only one pair in the state k . Because of the Pauli principle we have $\mathcal{C}_k + \mathcal{C}_k \mathcal{P}_k = 1$ and

$$\mathcal{C}_k = 1 - \sigma. \quad (61)$$

Next, we define the vacuum to vacuum transition amplitude by an intermediate effective action $\mathcal{A}(\text{vac} - \text{vac}) = \exp(iS_{eff})$. The vacuum to vacuum probability can be then written as $|\mathcal{A}(\text{vac} - \text{vac})|^2 = \exp(-2\text{Im} S_{eff}) = \prod_k \mathcal{C}_k$. It follows from (61) that

$$\exp(-2\text{Im} S_{eff}) = \exp \left[\sum_k \ln(1 - \sigma) \right] \quad (62)$$

and, consequently,

$$2\text{Im} S_{eff} = \Gamma = - \int \frac{dk}{2\pi a(t)} \ln(1 - \sigma). \quad (63)$$

By expanding the quantity $\ln(1 - \sigma)$, we obtain

$$\Gamma = \int \frac{dk}{2\pi a(t)} \sum_{n=1} \frac{1}{n} \exp \left[-\frac{2\pi}{H} \left(\mathcal{M} - \epsilon \frac{eE_0}{H} \right) \right]. \quad (64)$$

In the flat Minkowski space-time the integration over k can be carried out by making the change $\int dk \rightarrow eE_0 \int dt = eE_0 T$, which is consistent with the classical equation of motion (i.e. $\frac{dk}{dt} = eE_0$). In de-Sitter space, taking into account that the semiclassical approximation is valid as soon as the classical action is large, it follows from equation (A3) that the "in" and the "out" states behave like exponential functions with the classical action as exponent for $|\eta| \gg \frac{\mathcal{M}^2}{eE_0 k}$ and $|\eta| \ll \frac{\mathcal{M}^2}{eE_0 k}$ respectively. This means that the particle creation happens around the time $\eta \approx -\frac{\mathcal{M}^2}{eE_0 k}$ or, equivalently, particles having a wave vector k are created on the average at the time $\eta \approx -\frac{\mathcal{M}^2}{eE_0 k}$. Then, in de-Sitter space, we have to set $k = -\frac{\mathcal{M}^2}{eE_0 \eta} = \frac{\mathcal{M}^2 H}{eE_0} e^{Ht}$. Here t has to be understood as the time at which a particle is created with the wave vector k . Besides, at any time t , particles are produced with a wave vector in the range

$$-k_0 + \frac{H\mathcal{M}^2}{eE_0} e^{Ht} \leq k \leq k_0 + \frac{H\mathcal{M}^2}{eE_0} e^{Ht}, \quad (65)$$

where k_0 is a positive constant. For the case when $\frac{H\mathcal{M}^2}{eE_0} e^{Ht} < k_0$ we can write Γ in the form

$$\begin{aligned}\Gamma &= \sum_{n=1} \frac{1}{n} \exp \left(-\frac{2\pi n}{H} \mathcal{M} \right) \int_0^{k_0 + \frac{H\mathcal{M}^2}{eE_0} e^{Ht}} \frac{dk}{2\pi a(t)} \exp \left(2\pi n \frac{eE_0}{H^2} \right) \\ &+ \sum_{n=1} \frac{1}{n} \exp \left(-\frac{2\pi n}{H} \mathcal{M} \right) \int_{-k_0 + \frac{H\mathcal{M}^2}{eE_0} e^{Ht}}^0 \frac{dk}{2\pi a(t)} \exp \left(-2\pi n \frac{eE_0}{H^2} \right).\end{aligned}\quad (66)$$

The probability of particle creation per unit of time will be then given by

$$\frac{d\Gamma}{dt} = \frac{m^2 H^2 + e^2 E_0^2}{e E_0} \sum_{n=1} \frac{1}{n} \exp\left(-\frac{2\pi n}{H} \mathcal{M}\right) \sinh\left(2\pi n \frac{e E_0}{H^2}\right). \quad (67)$$

This procedure allows us to write the number of created fermions per unit of time in the form

$$\frac{dN}{dt} = \frac{m^2 H^2 + e^2 E_0^2}{\pi e E_0} \sinh\left(2\pi \frac{e E_0}{H^2}\right) \exp\left(-\frac{2\pi}{H} \mathcal{M}\right), \quad (68)$$

which is the same as the first term in the Schwinger-like series (67).

Notice that $\frac{d\Gamma}{dt}$ and $\frac{dN}{dt}$ are finite when the electric field is present. It is obvious also that $\frac{dN}{dt}$ and $\frac{d\Gamma}{dt}$ converge when $E_0 \rightarrow 0$. The introduction of an electric field plays then an important role in the theory. This is a sort of regularization. When t is large we have $\frac{H \mathcal{M}^2}{e E_0} e^{Ht} > k_0$ and, consequently, the probability of particle creation per unit of time and the pair production rate become

$$\frac{d\Gamma}{dt} = \frac{m^2 H^2 + e^2 E_0^2}{e E_0} \sum_{n=1} \frac{1}{n} \exp\left[-\frac{2\pi n}{H} \left(\mathcal{M} - \frac{e E_0}{H}\right)\right] \quad (69)$$

and

$$\frac{dN}{dt} = \frac{m^2 H^2 + e^2 E_0^2}{\pi e E_0} \exp\left[-\frac{2\pi n}{H} \left(\mathcal{M} - \frac{e E_0}{H}\right)\right]. \quad (70)$$

As is done in the previous section when $H \rightarrow 0$ equation (67) reduces to

$$\frac{d\Gamma}{dt} = \frac{e E_0}{2\pi} \sum_{n=1} \frac{1}{n} \exp\left(-n\pi \frac{m^2}{e E_0}\right), \quad (71)$$

which is the same as equation (64) in [49].

V. CONCLUDING REMARKS

In conclusion, we have studied in this paper the influence of an electric field on the creation of spin- $\frac{1}{2}$ particle pairs in the (1+1) inflationary de-Sitter space-time. We have found exact solutions for the corresponding Dirac equation. This is accomplished by the use of an unitary transformation which allows us to obtain a simple second-order differential equation for each component of the Dirac spinor. The charge symmetry between positive and negative frequency modes permitted us to find exact and analytic expressions for the Bogoliubov coefficients without need to normalize the wave functions. Then the pair creation probability and the density of created fermions are calculated by applying the canonical method based on Bogoliubov transformation connecting the "in" with the "out" states. From these results we were able to extract the imaginary part of the Schwinger effective action.

We have found that equation (53) in the present paper differs from equation (45) in [29]. This is due to a mistake in equation (42) in [29] which leads to incorrect results. Furthermore, equation (46) in [29] is obtained by considering the case when $|\frac{\mathcal{M}}{H} \pm \frac{e E_0}{H^2}| \gg 1$ and using the approximation $\sinh \pi \left(\frac{\mathcal{M}}{H} + \frac{e E_0}{H^2}\right) \approx \sinh \pi \left(\frac{\mathcal{M}}{H} - \frac{e E_0}{H^2}\right)$ which is valid only for a weak electric

field. When the effect of the electric field is significant (i.e. the case of strong field) the good approximation can be written as $\sinh \pi \left(\frac{\mathcal{M}}{H} + \frac{eE_0}{H^2} \right) \approx \exp \left(2\pi \frac{eE_0}{H^2} \right) \sinh \pi \left(\frac{\mathcal{M}}{H} - \frac{eE_0}{H^2} \right)$.

We conclude through this study that a constant electric field amplifies the creation of fermions with positive wave number k and minimizes it in the opposite direction. The inverse is true for antiparticles. In addition we have shown that the creation of light particles is possible in the presence of the electric field and thermal spectrum of particle creation will be obtained only when \mathcal{M} satisfy the adiabatic condition $\frac{\mathcal{M}}{H} \gg 1$. Another important result is that the creation of massless particles with conformal coupling is impossible even if an electric field is present.

Let us note at the end of this work that the method used to solve the Dirac equation is useful not only in the present case but also for all situations where $A_z(\eta)$ has the same dependence on η as $a(\eta)$. This encourages us to consider more realistic cases and especially the (3+1) dimensional problem.

Appendix A: The relativistic Hamilton-Jacobi equation

The "in" and "out" states are chosen according to the asymptotic behavior of the semi-classical solutions constructed by the use of WKB approximation starting from the solutions of the relativistic Hamilton-Jacobi (H-J) equation. It is widely known that in the adiabatic condition, i.e. $\mathcal{M} \gg H$, this approximation proved most fruitful in studying the particle creation either for scalar particles or for fermions. In this appendix we give the analytic solution of the H-J equation. The general form of the H-J equation is given by

$$g^{\mu\nu} (\partial_\mu S + eA_\mu) (\partial_\nu S + eA_\nu) - m^2 = 0 \quad (\text{A1})$$

For the case of dS_2 space with constant electric field the classical action S can be decomposed as

$$S = G(\eta) + kx, \quad (\text{A2})$$

where the time dependent part $G(\eta)$ satisfies the following equation

$$\frac{\partial}{\partial \eta} G(\eta) = \pm \sqrt{\left(k + \frac{eE_0}{H^2 \eta} \right)^2 + \frac{m^2}{H^2 \eta^2}}. \quad (\text{A3})$$

By the use of the well-known integral [43]

$$\begin{aligned} \int \frac{\sqrt{\alpha + \beta x + \gamma x^2}}{x} dx = & \sqrt{\alpha + \beta x + \gamma x^2} - \sqrt{\alpha} \operatorname{arcsinh} \left(\frac{2\alpha + \beta x}{x \sqrt{4\alpha\gamma - \beta^2}} \right) \\ & + \frac{\beta}{2} \frac{1}{\sqrt{\gamma}} \operatorname{arcsinh} \left(\frac{2\gamma x + \beta}{\sqrt{4\alpha\gamma - \beta^2}} \right), \end{aligned} \quad (\text{A4})$$

where α , β and γ are real numbers, with $\gamma > 0$ and $4\alpha\gamma - \beta^2 > 0$, we obtain

$$\begin{aligned}
G(\eta) = & \pm \sqrt{k^2 \eta^2 + 2k \frac{eE_0}{H^2} \eta + \frac{\mathcal{M}^2}{H^2}} \\
& \mp \frac{\mathcal{M}}{H} \ln \left(\frac{\frac{\mathcal{M}^2}{H^2} + k \frac{eE_0}{H^2} \eta}{\frac{m}{H} k \eta} + \sqrt{\left(\frac{\frac{\mathcal{M}^2}{H^2} + k \frac{eE_0}{H^2} \eta}{\frac{m}{H} k \eta} \right)^2 + 1} \right) \\
& \pm \frac{eE_0}{H^2} \ln \left(k\eta + \frac{eE_0}{H^2} + \sqrt{\left(k\eta + \frac{eE_0}{H^2} \right)^2 + \left(\frac{m}{H} \right)^2} \right). \tag{A5}
\end{aligned}$$

Notice that at $\eta \rightarrow 0$, we have $G(\eta) \approx \pm \mathcal{M}t + \text{a Constant}$.

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